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ULTRASONIC REFLECTION SPECTRUM ANALYSIS FOR NDT OF PERIODIC LAYERED MEDIA

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This report investigates in detail the normal incidence acoustic reflection spectra from periodic, layered media and emphasizes applications of the results to nondestructive testing (NDT) of composite materials.

In contrast to a previous study which derived a number of exact but iterative techniques for predicting the acoustic spectrum for a given laminate, this study derives a number of general, closed form expressions for both finite

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and infinite periodic laminae which defines measurable properties independent of the number laminae.

Using these results, composite materials are modeled as two component laminae with one component being a thin interface layer. Interesting features of this model are low frequency velocities which are dependent on composition but not lamina thickness and an expression for boundary layer thickness in terms of band gaps in the frequency spectrum.

Comparisons of experimental and theoretical results show that although effects of periodic structures are measurable ultrasonically, other effects associated with material dispersion and inconsistency in material properties make it difficult to get consistent ultransonic measurements from some graphite epoxy composites.

Unless more uniformity in material properties can be obtained, the full potential of ultransonic inspection techniques cannot be realized.

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INTRODUCTION

A number of techniques have been developed for modeling the propagation of ultrasonic waves in layered media,[1,2,3] and a number of authors have also discussed the case for which such media are infinitely periodic in space[1,4,5]. However, none of these authors has fully treated the case of reflection and transmission through finite periodic media. Brillouin[4] has indicated these cases might be treated whenever boundary conditions are such that no waves are reflected from the extremity of the periodic medium, but has not worked out this problem in detail.

In this report a general technique for treating arbitrary finite periodic media for all boundary conditions is derived and the interesting case in which the ambient media at the two boundaries have identical acoustic properties is discussed in detail.

This generalization is accomplished by taking the iterative equations derived in reference[1] and reformulating them in such a manner as to represent each iteration as matrix multiplications. A matrix diagonalization technique is then used to produce a closed form expression for an arbitrary number of iterations "q" corresponding to a laminate of "q" layers.

The resulting formulae rigorously confirm many of the empirical observations of reference[1] relating to the spacings of thickness resonances and the positions of forbidden frequency bands.

THEORY

A. Wave Propagation in Layered Media

Techniques have been reported[2] for calculating the reflection and transmission coefficient of layered media when the transmission and reflection coefficients at each of the interfaces are known.

These equations may be applied to an array of n laminae of the type shown in figure 1. The medium to the right of the first interface is assumed to be infinite in extent as is the medium to the left of the n^{th} interface. In general a given medium i has a thickness d_i, a velocity C_i and has reflection and transmission coefficients r_i and t_i associated with waves incident from the left upon its left interface and has frequency dependent reflection and transmission coefficients R_i and T_i associated with the entire stack of laminae to the right of that interface.

Applying the equations of reference 1 to the laminate of figure 1 gives the following results:

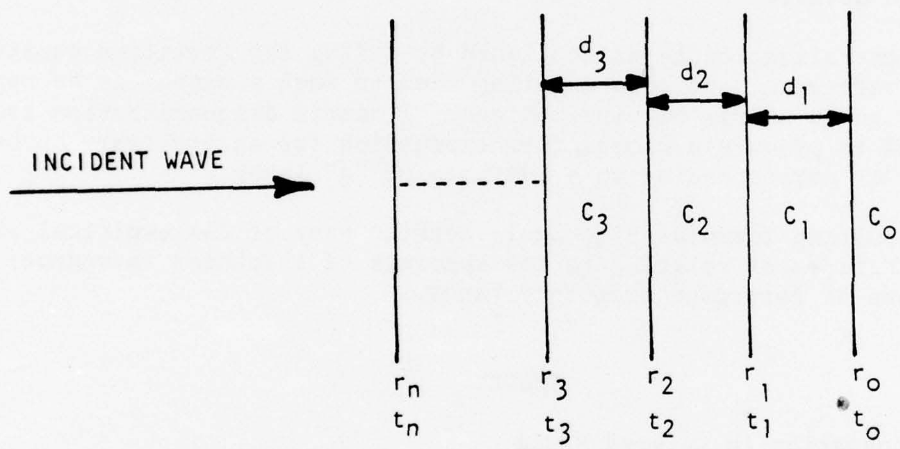

FIGURE 1. Plane Wave Incident Upon Layered Media

$$R_0 = r_0 \qquad T_0 = t_0 \tag{1}$$

$$R_1(\omega) = \frac{r_1 + r_0 \exp(-2i\omega d_1/C_1)}{1 + r_1 r_0 \exp(-2i\omega d_1/C_1)} \tag{2}$$

$$T_1(\omega) = \frac{t_1 t_0 \exp(-i\omega d_1/C_1)}{1 + r_1 r_0 \exp(-2i\omega d_1/C_1)} \tag{3}$$

and in general

$$R_n(\omega) = \frac{r_n + R_{n-1}(\omega) \exp(-2i\omega d_n/C_n)}{1 + r_n R_{n-1}(\omega) \exp(-2i\omega d_n/C_n)} \tag{4}$$

$$T_n(\omega) = \frac{t_n T_{n-1}(\omega) \exp(-i\omega d_n/C_n)}{1 + r_n R_{n-1}(\omega) \exp(-2i\omega d_n/C_n)}. \tag{5}$$

In order to make these equations more tractable it is desirable to transform them from an iterative into an explicit form; therefore, the following definitions are made:

$$a_n = r_n + R_{n-1} \exp(2i\phi_n) \tag{6a}$$

$$\phi_n = -\omega d_n/C_n \tag{6b}$$

$$b_n = 1 + r_n R_{n-1} \exp(2i\phi_n) \tag{6c}$$

from which it follows that

$$R_n = a_n/b_n \tag{7}$$

but

$$R_n = \frac{r_n b_{n-1} + \exp(2i\phi_n) a_{n-1}}{b^{n-1} + r_n \exp(2i\phi_n) a_{n-1}} \tag{8}$$

Clearly the two quantities a_n and b_n may then be written as a vector and equation 8 becomes:

$$\begin{pmatrix} a_n \\ b_n \end{pmatrix} = \begin{pmatrix} \exp(2i\phi_n) & r_n \\ r_n\exp(2i\phi_n) & 1 \end{pmatrix} \begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} \tag{9}$$

B. Periodic Layered Media

1. Reflectance Equations

Since only the ratio a_n/b_n is of interest a more symmetrical form may be used;

$$\begin{pmatrix} A_n \\ B_n \end{pmatrix} = \begin{pmatrix} \exp(i\phi_n) & r_n\exp(-i\phi_n) \\ r_n\exp(i\phi_n) & \exp(-i\phi_n) \end{pmatrix} \begin{pmatrix} A_{n-1} \\ B_{n-1} \end{pmatrix} \tag{10}$$

which immediately implies,

$$\begin{pmatrix} A_n \\ B_n \end{pmatrix} = \left[\prod_{m=1,n} \begin{pmatrix} \exp(i\phi_m) & r_m\exp(-i\phi_m) \\ r_m\exp(i\phi_m) & \exp(-i\phi_m) \end{pmatrix} \right] \begin{pmatrix} r_0 \\ 1 \end{pmatrix} . \tag{11}$$

This form is somewhat suggestive of the result of Brekhovskikh[3]; however, it has the advantage that it contains fewer implicit operations.

For periodic structures expression (11) takes the form.

$$\begin{pmatrix} A_n \\ B_n \end{pmatrix} = \left[\prod_{m=1,\ell} \begin{pmatrix} \exp(i\phi_m) & r_m\exp(-i\phi_m) \\ r_m\exp(i\phi_m) & \exp(-i\phi_m) \end{pmatrix} \right]^{\frac{n}{\ell}} \begin{pmatrix} r_0 \\ 1 \end{pmatrix} \tag{12}$$

$$\begin{pmatrix} A_{q\ell} \\ B_{q\ell} \end{pmatrix} = Mp^{q} \begin{pmatrix} r_0 \\ 1 \end{pmatrix} \ ; \text{ where } q = \frac{n}{\ell} \tag{12a}$$

n is the total number of layers in the structure, ℓ is the number of layers in one period of the structure, $\frac{n}{\ell}$ is the number of periods in the laminate, and M_p is the product matrix in expression 12.

Since the product matrix in (12) will not in general be hermitean, an expression for a 2x2 matrix raised to a power is required. For any matrix M it can readily be shown that for

$$M_p = \begin{pmatrix} e & f \\ g & h \end{pmatrix} \tag{13}$$

$$M_p \cdot M_p = (e+h)\ M + [2+fg-eh]\ \mathbb{1} \tag{14}$$

where $\mathbb{1}$ is the unit matrix

Since multiplying M by a constant factor will not affect the value of R_n the matrice $M = \dfrac{M_p}{\det(M_p)}$ may be used.

$$M \cdot M = 2\alpha M + \mathbb{1} \tag{15}$$

where $\alpha = \dfrac{e+h}{2(fg-eh)}$

It is clear from (15) that the product M^N **fg-eh** must be of the form

$$M^N = \alpha_N M + \beta_N \cdot \mathbb{1} \tag{16}$$

This may be evaluated for a specific case by repeated application of expression (15) or by using the matrix relation

$$\begin{pmatrix} \alpha_N \\ \beta_N \end{pmatrix} = \begin{pmatrix} 2\alpha & 1 \\ 1 & 0 \end{pmatrix}^{N-1} \begin{pmatrix} \alpha_0 \\ \beta_0 \end{pmatrix} = \begin{pmatrix} 2\alpha & 1 \\ 1 & 0 \end{pmatrix}^{N-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{17}$$

The 2x2 matrix in (17) is symmetric and has eigenvalues $\lambda = \alpha \pm \sqrt{\alpha^2+1}$ and eigenvectors $(1, \alpha + \sqrt{\alpha^2+1})$ and $(1, -\alpha - \sqrt{\alpha^2+1})$. Then for α real this matrix may be diagonalized as follows:

$$\begin{pmatrix} 2\alpha & 1 \\ 1 & 0 \end{pmatrix}^N = \frac{-(\alpha^2+1)^N}{(\alpha - \sqrt{\alpha^2+1})} \begin{pmatrix} (\alpha-\sqrt{\alpha^2+1})^{N+1} - (\alpha+\sqrt{\alpha^2+1})^{N+1} & (\alpha-\sqrt{\alpha^2+1})^N - (\alpha+\sqrt{\alpha^2+1})^N \\ (\alpha-\sqrt{\alpha^2+1})^N - (\alpha+\sqrt{\alpha^2+1})^N & (\alpha-\sqrt{\alpha^2+1})^{N-1} - (\alpha+\sqrt{\alpha^2+1})^{N-1} \end{pmatrix}$$

$$M^N = \frac{(\alpha^2+1)^N}{(\alpha-\sqrt{\alpha^2+1})} \left[\left((\alpha-\sqrt{\alpha^2+1})^{N+1} - (\alpha+\sqrt{\alpha^2+1})^{N+1} \right) M + \mathbb{1} \left((\alpha-\sqrt{\alpha^2+1})^N - (\alpha+\sqrt{\alpha^2+1})^N \right) \right] \tag{18}$$

2. Travelling Wave Regime

As an alternative to applying expression (18) we may diagonalize the matrix in 12a directly. The form of this matrix is easily shown to be,

$$M_p = \begin{pmatrix} M_{11} & M_{21}^* \\ M_{21} & M_{11}^* \end{pmatrix} \tag{19}$$

In the most general case such a matrix will have two distinct eigenvalues $\lambda\pm$ given by

$$\lambda\pm = 1/2 \left[M_{11} + M_{11}^* \pm \sqrt{(M_{11}+M_{11}^*)^2 - 4[|M_{11}|^2 + |M_{21}|^2]} \right] \tag{20a}$$

for $(M_{11} + M_{11}^*)^2 < 4\ [|M_{11}|^2 + |M_{21}|^2]$, (20b)

which is the range of periodic solutions, $\lambda\pm$ take the form

$$\lambda\pm = \rho\ e^{\pm i\psi} \tag{20c}$$

where $\cos\psi = \dfrac{M_{11} + M_{11}^*}{2\rho}$ (20d)

and $\rho^2 = [|M_{11}|^2 + |M_{21}|^2]$ (20e)

and from (12) $\rho^2 = \pi_m = 1,\ell\ (1-r^2_m)$ (20f)

Corresponding to these two eigenvalues two non-orthogonal eigenvectors may be constructed of the form

$$V\underline{+} = \begin{pmatrix} -M_{21} \\ M_{11} - \lambda\pm \end{pmatrix} \tag{21}$$

Suppressing the ℓ subscript 12a may be written,

$$\begin{pmatrix} A_q \\ B_q \end{pmatrix} = M_p^q \begin{pmatrix} r_0 \\ 1 \end{pmatrix} = \mu\rho e^{+i_q\psi} V+ \;+\; \nu\rho e^{-i_q\psi} V- \tag{22}$$

where μ and ν are the components of the vector $\begin{pmatrix} r_0 \\ 1 \end{pmatrix}$ with respect to the independent basis vectors V+ and V-.

Again as in (10), ignoring extraneous proportionality factors R_q may be expressed in the form

$$R_q = \left(\frac{A_q{}^1}{B_q{}^1} \right) \tag{23}$$

where $\begin{pmatrix} A_q{}^1 \\ B_q{}^1 \end{pmatrix} = \begin{pmatrix} -\lambda^q + M_{21} & -\lambda^q - M_{21} \\ \lambda^q + (M_{11}-\lambda_+) & \lambda_-^q (M_{11}-\lambda_-) \end{pmatrix} \begin{pmatrix} M_{11}-\lambda & M_{21} \\ \lambda_+ - M_{11} - M_{21} \end{pmatrix} \begin{pmatrix} r_0 \\ 1 \end{pmatrix}$

In order to illustrate the **nature** of one such solution let us consider the important case suggested by Brillouin in which $r_0 = 0$. This may be realized by placing an infinite medium on the right side of the array which is identical to the last medium on the right side of the array (see figure 1).

Solving for the coordinate in the eigenvector space gives the simultaneous equation

$$-\mu\, M_{21} \qquad -\nu\, M_{21} = 0$$

$$\mu(M_{11}-\lambda_+) + \nu(M_{11}-\lambda_-) = 1.$$

Which imply

$$\nu = -\mu$$

$$\mu = [\lambda_- - \lambda_+]^{-1} = [2\rho i \sin\psi]^{-1}.$$

In fact the value of μ is of no consequence since it cancels in the final expression and we have

$$R_q = \frac{-(\lambda^q{}_+ - \lambda^q{}_-)\ M_{21}}{\lambda_+{}^q(M_{11}-\lambda_+) - \lambda^q{}_-(M_{11}-\lambda_-)} \tag{24a}$$

$$= \frac{M_{12}\sin_q\psi}{-M_{11}\sin_q\psi + \rho^{\sin}(qH)\psi}$$

$$R_q = \frac{M_{21}}{\rho^{\sin}\psi\cot_q\psi - (M_{11}-\rho^{\cos}\psi)} \tag{24b}$$

Since M_{11} and M_{12} are sums of products of bounded periodic functions R will will have zeroes wherever cot $\psi \to \infty$

$$\psi = \frac{n_\pi}{q} \quad \text{for n an integer} \neq 0 \tag{25}$$

Then assuming that ψ is a monitor function of frequency R_q will have q -1 minima for which R_p=0 between ψ=ω=0 and ψ=π. For cases in which $\frac{n}{q}$ is an integer (24b) has an indeterminant form in the denominator and each case must be considered carefully.

In section (5) it is demonstrated that this result may also be obtained for a more general set of boundary conditions in which the ambient media bordering the array from the right and left are identical.

3. Non-Travelling Wave Solutions

For $(M_{11} + M_{11}{}^*)^2 > 4\ (|M_{11}|^2 + |M_{21}|^2)$

again set $\rho^2 = [|M_{11}|^2 + |M_{21}|^2]$

and $\lambda_\mp = \rho_e{}^{\gamma\pm}$

where $\gamma_\pm = \log_e\left(\dfrac{M_{11}+M_{11}{}^* \pm \sqrt{(M_{11}+M_{11}{}^*)^2-4\rho}}{\rho^2}\right)$

It is a straightforward substitution to show that $e^{\gamma+} = (e^{\gamma-})^{-1}$. Then letting $\gamma = \gamma^+$ the expression for the eigenvalues becomes,

$$\lambda_\pm = \rho^{1/2}e^{\pm\gamma}. \tag{26}$$

Together with (24a) this gives

$$R_q = \frac{-(\sinh(_q\gamma))\ M_{21}}{M_{11}(\sinh(_q\gamma)) - (\sinh(_{q+1})\gamma)}$$

$$R_q = \frac{M_{21}}{\rho\cosh\gamma + \rho\sinh\gamma\ \coth\gamma - M_{11}} \tag{27}$$

The hyperbolic functions are not periodic and over a range of monotonically increasing γ they should exhibit at most one maximum or minimum.

4. Phase Velocity

The phase velocity of sound for a finite plate in a uniform ambient medium has been shown to be,

$$C = \frac{2df}{n} \tag{28}$$

where f is the frequency of the n^{th} thickness resonance from a plate laminate of thickness d.

It will now be shown that (28) when used as a definition for phase velocity allows a generalization of this concept to periodic layered media. It is by no means intuitively obvious that phase velocity in a laminate is a material property uniquely defined by (28), since the positions of thickness resonances could be affected by boundary conditions in a thickness dependent manner. This in fact appears to be the case whenever, there exists at the far interface, a non zero reflection amplitude off a medium which is different from the incident medium. Clearly this limits the measurement situations in which the definition (28) can be applied. The definition is further limited in that it can only be applied to the determination of velocity at particular frequency intervals. However this should not limit the validity of the definition as long as it is not inconsistent for laminates of different thicknesses. It will now be demonstrated that this is in fact true whenever the relations (25) and (20d) are satisfied.

From equation (24b) such frequency resonances occur in periodic laminates whenever:

$$\psi = \frac{n\pi}{q} \tag{25}$$

where ψ is implicitly defined by the relationship

$$\cos\psi = \frac{M_{11} + M_{11}^*}{2\rho} \tag{20d}$$

Since (25) and (28) are both functions of $\left(\frac{q}{n}\right)$ and therefore not dependent on d and since ψ is also independent of d then,

$$C = \frac{2\pi\ Df}{\psi(f)} = \frac{\omega D}{\psi(f)} \tag{29}$$

This yields the general result that the phase velocity of sound in any periodic medium as defined by (29) is independent of the thickness of that medium whenever (25) and (20d) are satisfied. The next section will explore some general conditions for which these relations hold.

5. The Effects of an Ambient Medium

In sections (2) and (4) above an expression was derived for reflectance and phase velocity for any periodic media for which the acoustic properties of the outer boundary layers match those of the ambient medium. In this section we will generalize those results to periodic media for which the acoustic properties of the outer boundary layers are identical to each other but not necessarily to those of the ambient medium. This case is approximated by any composite laminate emersed in a water bath for ultrasonic interrogation.

In general the matrix representing such a situation can be written in the form

$$M_p^q \begin{pmatrix} r' \\ 1 \end{pmatrix} = \overline{M}_s M_o^{-1} (M_p)^q M_s \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{30}$$

where M_p is the product matrix for one laminate period
q is the number of laminate periods
M_o is the matrix for the outer layer at the laminate interior
$\overline{M}_s$ is the matrix for the outer layer at the incident interface
M_s is the matrix for an arbitrary layer of incident medium.

(30) can readily be transformed into the form,

$$M_p^q \begin{pmatrix} r^{1'} \\ 1 \end{pmatrix} = \overline{M}_s M_o^{-1} M_s \; (M_s^{-1} M_p M_s)^q \; \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} e^{i\phi_1} & \gamma e^{-i\phi_1} \\ \gamma e^{i\phi_1} & e^{-i\phi_1} \end{pmatrix} \begin{pmatrix} e^{-i\phi_1} & \\ & e^{i\phi_1} \end{pmatrix} \begin{pmatrix} 1 & -\gamma \\ -\gamma & 1 \end{pmatrix} \left[\begin{pmatrix} 1 & -\gamma \\ -\gamma & 1 \end{pmatrix} \begin{pmatrix} M_{11} & M_{21}^* \\ M_{21} & M_{11}^* \end{pmatrix} \begin{pmatrix} 1 & \gamma \\ \gamma & 1 \end{pmatrix} \right]^q \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

where ϕ_1 is the phase shift associated with the outer layer and γ is the reflection amplitude coefficient for a wave reflected from the outer medium.

$$M_p^9 = (1-\gamma^2)^{-1} \begin{pmatrix} M_{11}-\gamma^2 M_{11}^* & \gamma(M_{11}-M_{11})^* + M_{21}^* - \gamma^2 M_{21} \\ \gamma(M_{11}^* - M_{11}) + M_{21} - \gamma^2 M_{21} & M_{11}^* - \gamma^2 M_{11} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{31}$$

which from 20d yields,

$$\cos\psi = \frac{M_{11} + M_{11}^*}{2\rho} \tag{32}$$

This implies that the phase velocity as defined in section (4) applies to a large class of periodic layered media.

C. Two Component Media (A Composite Approximation)

1. Reflectance for Travelling Waves

Consider the periodic medium of figure 2. Applying expressions (12) and (12a) gives the expression,

$$M_p = \begin{pmatrix} e^{i\phi_B} & 0 \\ 0 & e^{-i\phi_B} \end{pmatrix} \begin{pmatrix} e^{i\phi_A} & -re^{-i\phi_A} \\ -re^{-i\phi_A} & e^{-i\phi_A} \end{pmatrix} \begin{pmatrix} e^{i\phi_B} & re^{-i\phi_B} \\ re^{i\phi_B} & e^{-i\phi_B} \end{pmatrix} \tag{33}$$

$$= \begin{pmatrix} e^{i(\phi_A+2\phi_B)} - r^2 e^{i(2\phi_B-\phi_A)} & r(e^{i\phi_A} - e^{-i(\phi_A)}) \\ r(e^{-i\phi_A} - e^{i\phi_A}) & e^{-(2\phi_B+\phi_A)} - r^2 e^{i(\phi_A-2\phi_B)} \end{pmatrix}$$

from expression 20d

$$\cos\psi = \frac{\cos(2\phi_B+\phi_A) - r^2\cos(2\phi_B-\phi_A)}{(1-r^2)} \tag{34}$$

From 24b for a laminate of q periods,

$$R_q = \frac{ir\sin\phi_A}{\rho\sin\psi\cot_q\psi - (e^{i\phi_A+2i\phi_B} - r^2 e^{i(2\phi_B-\phi_A)} - \rho\cos\psi)} \tag{35}$$

2. Velocity of Sound-Low Frequency Limit.

The use of expression (29) for velocity of sound requires the inversion of (34) which produces a rather unwieldy result; however in certain limits approximations can be made which produce tractable expressions. For $\phi_A+2\phi_B$ small (34) becomes

$$\cos\psi = 1 + \frac{1/2\ (4\phi_B{}^2+4\phi_A\phi_B+\phi_A{}^2) - r^2/_2(4\phi_B{}^2-4\phi_A\phi_B+\phi_B{}^2)}{1-r^2} \tag{36}$$

$$\simeq 1 + \frac{\psi^2}{2}$$

$$\psi = \left[(2\phi_B\phi_A)^2 + \frac{16r^2\phi_A\phi_B}{1-r^2} \right]^{1/2} \tag{37}$$

$$C = \frac{d_A + d_B}{\left[\left(2\frac{d_B}{C_B} + \frac{d_A}{C_A} \right)^2 + \frac{16r^2}{1-r^2}\frac{d_A d_B}{C_A C_B} \right]^{1/2}} \tag{38}$$

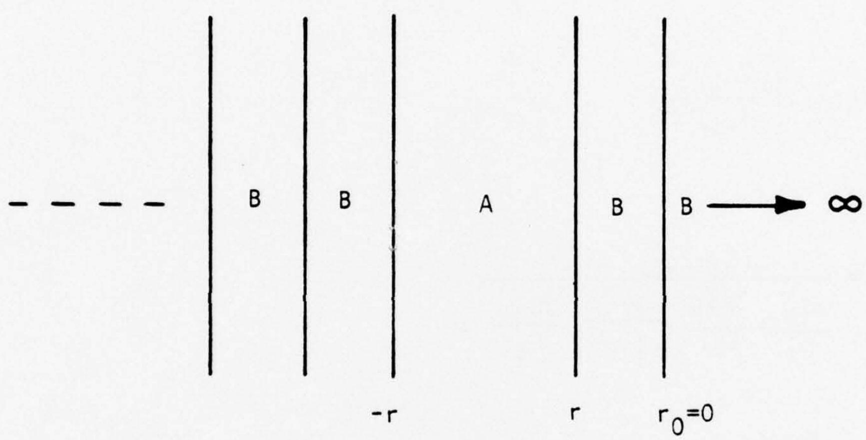

FIGURE 2. Two Component Layered Media

3. Velocity of Sound-Limit of Thin Boundary Layers and Low Frequency.

Again from (34),

$$\cos\psi = \frac{\cos(2\phi_B+\phi_A) - r^2 \cos(2\phi_B-\phi_A)}{(1-r^2)}$$

For $2\phi_B \ll \phi_A$

$$\cos\psi \approx \cos\phi_A + 2(\sin\phi_A)\phi_B \frac{(r^2-1)}{(r^2+1)} \tag{39}$$

for ω small

$$\cos\psi \approx \cos\left(\phi_A - 2\left(\frac{r^2-1}{r^2+1}\right)\phi_B\right)$$

From (30),

$$C = \frac{\omega D}{\psi} = \frac{\omega(d_A+2d_B)}{\frac{\omega d_A}{C_A} + \frac{2\omega d_B}{C_B} + \frac{2r^2}{(1-r^2)} \frac{2d_B\omega}{C_B}}$$

$$C = \frac{1+\left(\frac{2d_B}{d_A}\right)}{\frac{1}{C_A} + \left(\frac{2d_B}{d_A}\right)\frac{1}{C_B} + \frac{2r^2}{1-r^2}\left(\frac{2d_B}{d_A}\right)\frac{1}{C_B}} = \frac{d_A+2d_B}{\frac{d_A}{C_A} + \frac{2d_B}{C_B} + \frac{2r^2}{(1-r^2)} \frac{d_B}{C_B}} \tag{40}$$

The third term in the denominator of (40) describes the effect of internal reflections retarding the phase of the wave train passing through the laminate. The expression (40) will probably be useful for relating static and dynamic (i.e., ultrasonic) measurements of modulus at low frequencies. When r=0 the internal reflections disappear and (40) reduces to the "a priori" result

$$C_o = \frac{d_A+2d_B}{\frac{d_A}{C_A} + \frac{2d_B}{C_B}} \geq C \tag{41}$$

which is the usual expression for average velocity. It is of interest that both (40) and (41) are dependent only on the ratio of $d_A/2d_B$ (i.e., the percent composition) and not on specific thickness. For this reason (41) may be a reasonable approximation for the low frequency velocity of sound in materials

with randomly spaced layers such as directionally solidified eutectics.

4. Discontinuity in Phase Velocity for a Two Component Laminate with Thin Boundary Layers

From (34)

$$\cos\psi = \frac{\cos(2\phi_B+\phi_A)-r^2\cos(2\phi_B-\phi_A)}{(1-r^2)}$$

let $\phi_A >> 2\phi_B$ and $\phi_A' = \pi-\phi_A$

Then for $\phi_A \simeq \pi$ we have,

$$\cos\psi = -1+\frac{(2\phi_B{}^2-2\phi_A{}^1\phi_B+\frac{\phi_A'^2}{2})-r^2(2\phi_B{}^2+2\phi_A{}^1\phi_B+\frac{\phi_A'^2}{2})}{1-r^2}$$

$$\cos\psi = -1+2\phi_B{}^2+\frac{\phi_A'^2}{2}+\frac{2r^2+1}{r^2-1}\phi_A'\phi_B \tag{42}$$

In order to find the region of phase velocity discontinuity we set

$$2\phi_B{}^2+\frac{2r^2+1}{r^2-1}\phi_A'\phi_B+\frac{\phi_A'^2}{2}=0$$

which produces the condition

$$\phi_A' = -2\frac{r^2+1}{r^2-1}\phi_B\left[1\pm\sqrt{\left(\frac{r^2+1}{1-r^2}\right)^2-1}\right] \tag{43}$$

The approximate frequency and width of the discontinuity may now compute from (43)

$$\pi-\phi_A = -2\frac{r^2+1}{r^2-1}\phi_B\left[1\pm\sqrt{\left(\frac{r^2+1}{r^2-1}\right)^2-1}\right]$$

$$f_\pm = \left[\frac{2d_A}{C_A}+\frac{4d_B}{C_B}\left[\frac{r^2+1}{1-r^2}\right]\left[1\pm\sqrt{\left(\frac{r^2+1}{r^2-1}\right)^2-1}\right]\right]^{-1} \tag{44}$$

For r=0 this reduces to $f_o=\left[\frac{2d_A}{C_A}+\frac{4d_B}{C_B}\right]^{-1}$ which is the "a priori" first approximation to the thickness resonance frequency for the two layers A and B together and no forbidden gap exists.

Since $\frac{r^2+1}{1-r^2} \geq 1$ for all physically allowable values of r, is is clear that the onset of the first forbidden band is below f_o and since

$$\left(\frac{r^2+1}{1-r^2}\right) \left(1 \pm \sqrt{\left(\frac{r^2+1}{r^2-1}\right)} -1 \right) >0$$

the end of the forbidden band will always be below the thickenss resonance for the acoustically thickest layer

$$fOA = \frac{C_A}{2d_A} > f_{\pm} \tag{45}$$

For computational purposes when dealing with small discontinuities it is most convenient to use the relationship,

$$\Delta\left(\frac{1}{f}\right) = 8\,\frac{d_B}{C_B}\left[\frac{1+r^2}{1-r^2}\right]\left[\left(\frac{(1+r^2)}{(1-r^2)}\right)^2 -1\right]^{1/2} \tag{46}$$

which is derived directly from (44).

$$\text{If } \Omega(r) = \left[\frac{1+r^2}{1-r^2}\right]\left[\frac{(1+r^2)^2}{(1-r^2)^2} -1\right]^{1/2}$$

(46) becomes,

$$\frac{\Delta f}{f^2} = \frac{8d_B}{C_B}\,\Omega(r) \tag{47}$$

Since $\Omega(r)$ varies slowly with r (See figure 3) even a very rough estimate of r will allow a fairly accurate determination of the bond line thickness ($2d_B$). Conversely if the bond line thickness is known with reasonable accuracy, a very accurate determination of the value of r can be made.

NUMERICAL RESULTS

Water/Glass Arrays

Equation (25) is an important result since it allows the prediction of ultrasonic reflection minima for any periodic medium subject to the conditions established in section B-5 above. This prediction requires only that the function ψ be tabulated as a function of ω.

As a numerical check on the validity of Equation (25) and the results of section B-5 the reflectance spectrum for an array of water and glass laminae in in an ambient medium of ethyl alcohol was computed using the formulae (1) through (5). The total number of media involved were 11 and these constituted

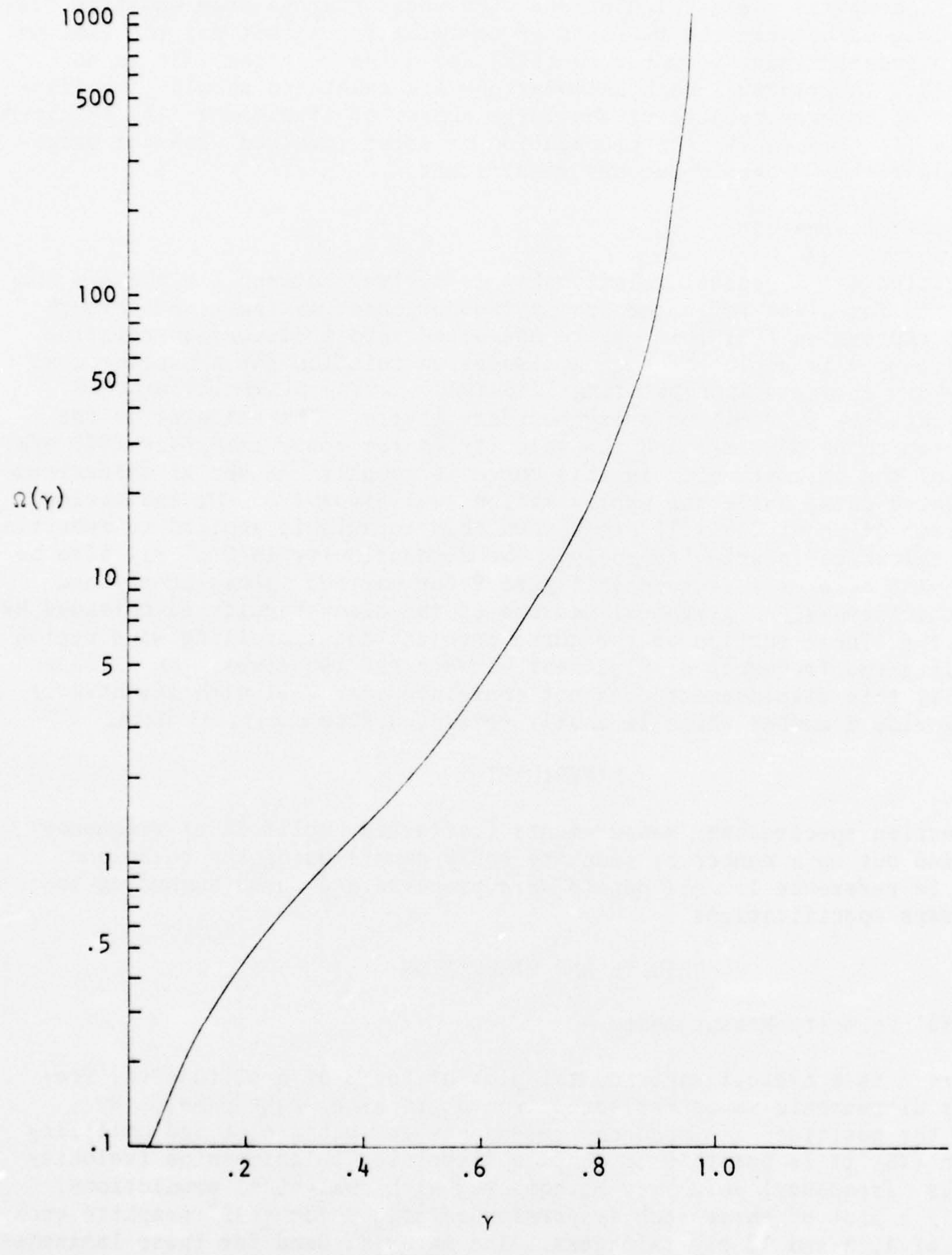

FIGURE 3. Ω(r) Versus r

a four period array (i.e., $q = 4$) in a non matching ambient medium.

Table I compares these calculations with ones obtained from equation (25). Agreement is good between the two sets of calculations except for the case $n=4$ which is an indeterminant form for eq (24b) for which relation (25) is no longer valid. In principal both calculations are exact and should give identical results; however because of the large number of iterations (11) required for formula (1) through (5) the propagation of error involved with the trigonometric algorithms probably becomes significant.

Graphite/Epoxy Laminates

In section C-1 a general relationship is derived between the phase ψ and the frequency for waves reflected from a two component laminate and through the use of expression (29) this may be converted into a dispersion relation $C=C(\omega)$. Figure 4 is a plot of such a dispersion relation for a hypothetical graphite epoxy laminate approximating 5208-T-300 having plies 178 microns thick separated by 9.90 micron epoxy boundary layers. The velocity in the plys is taken to be 3300 m/s and the velocity in the epoxy interface 2200 m/s. The width of the discontinuity in this curve is roughly .85 MHz as determined from tabulated data, while the approximation (46) gives 1.06 MHz indicating that an error of about 20% will occur when this formula is applied to measuring bond line thickness in graphite epoxy. The discontinuity in $C(\omega)$ may also be seen in a plot of ω vs ψ as seen in figure 5 for various values of ply and bondline thicknesses. A graphical measure of the discontinuity is obtained by extending the linear portion of the curve into the non-travelling wave region and measuring the frequency displacement between the two lines. As is shown by table II, this displacement does not correlate very well with Δf; however it does provide a number which is easily generated from empirical data.

EXPERIMENT

Reflection spectroscopy measurements (reflected amplitude vs frequency) were carried out on a number of graphite epoxy panels using the technique described in reference 1. All panels were prepared and cured according to manufacturers specifications.

RESULTS AND DISCUSSION

Experimental Velocity Measurements

Figure 6 is a typical experimental plot of Log_{10} of amplitude vs. frequency for ultransonic waves reflected from a graphite epoxy panel. By measuring the positions of frequency resonances on such a plot and utilizing expression (28) it is possible to compute dispersion relationships (velocity of sound vs. frequency) which may be compared with analytical predictions. Figure 7 is a plot of three such dispersion relations for $\pm$ 45° graphite epoxy laminates of 4, 8 and 12 ply thickness. The material used for these laminates was 5208-T-300 graphite epoxy with a nominal 5 mil. (127 micron) ply thickness.

The actual values of the laminate thicknesses are shown in table III along with the resultant average ply thicknesses. These values were used in the

TABLE I - COMPARISON OF PEAK POSITIONS FOR THE ULTRASONIC REFLECTANCE SPECTRUM OF A WATER/GLASS ARRAY AS COMPUTED BY TWO DIFFERENT TECHNIQUES.

PEAK NUMBER (n)	PEAK POSITION	
	FROM EQ 25	FROM EQS 1-5
1.	.289	.294
2	.554	.568
3	.755	.775
(4)	———	(.824)
5	2.267	2.26
6	2.397	2.39
7	2.519	2.52
(8)	———	———
9	4.42	4.38
10	4.53	4.50
11	4.65	4.62
(12)	———	———
13	6.33	6.29
14	6.49	6.52
15	6.72	6.74

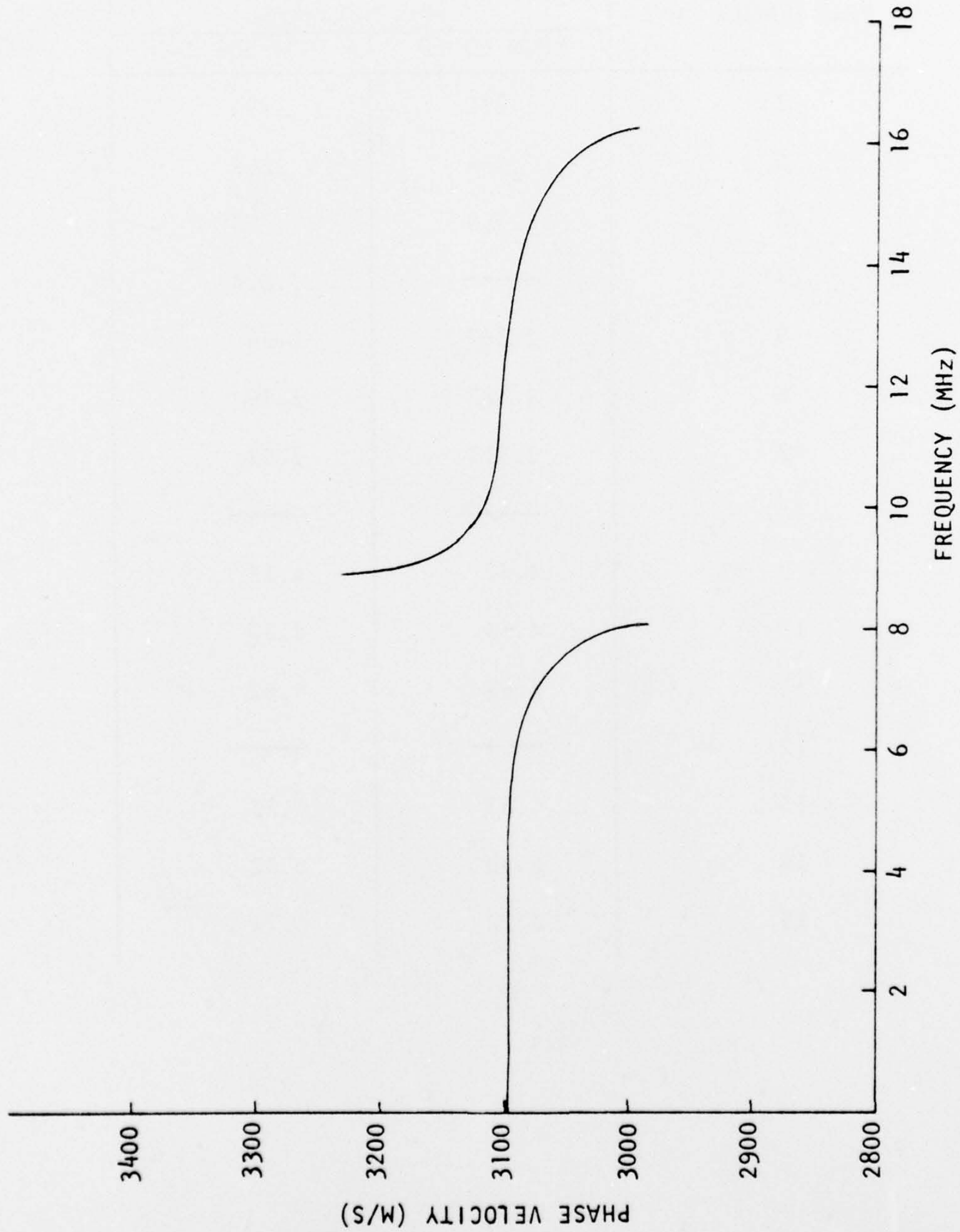

FIGURE 4. Dispersion Relation For Hypothetical Laminate

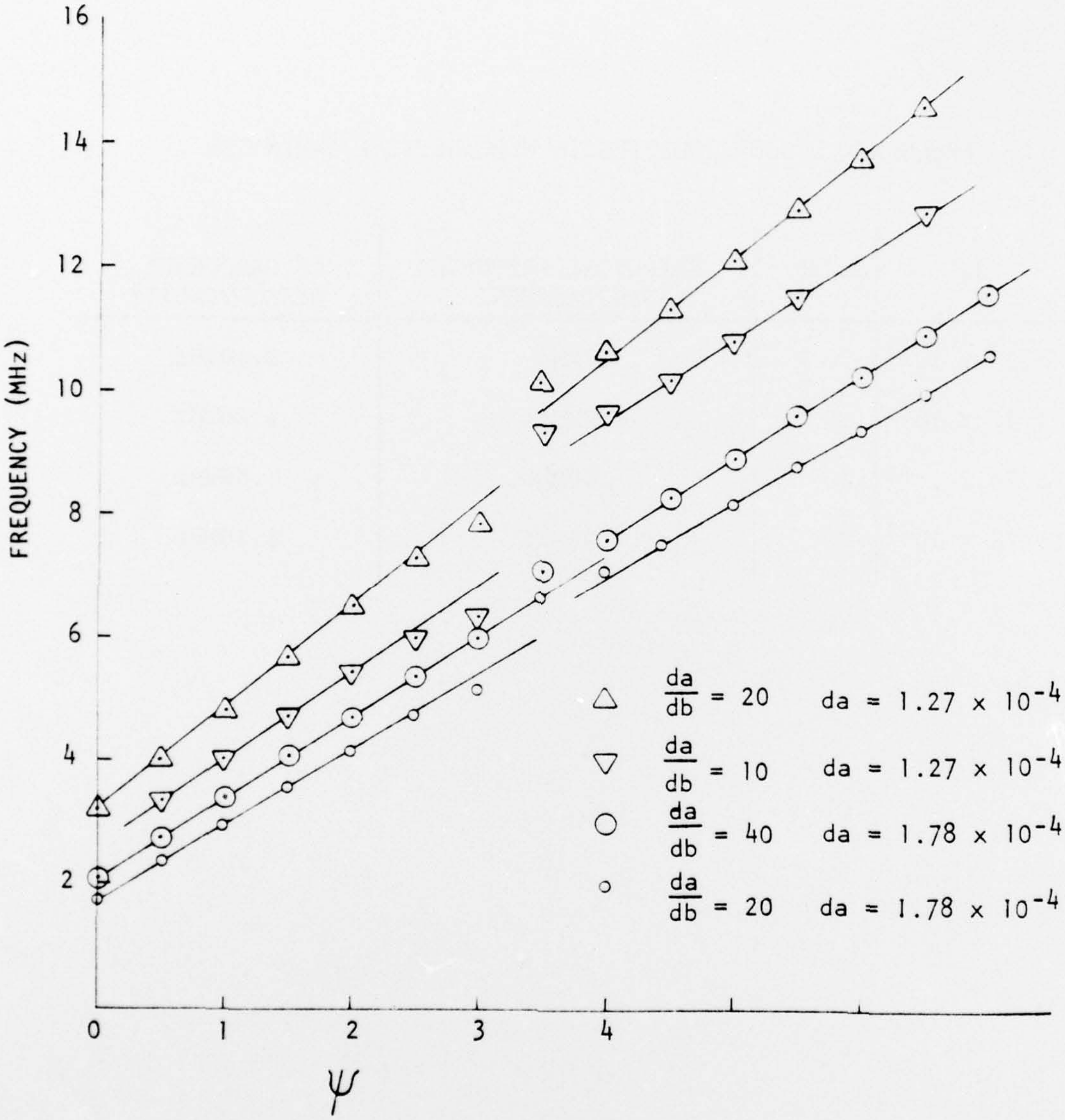

FIGURE 5. Dispersion Relations For Hypothetical Laminates of Varying Thickness

TABLE II - FREQUENCY DISCONTINUITIES IN HYPOTHETICAL LAMINATES

da	da/db	GRAPHICAL FREQUENCY DISPLACEMENT	ΔF FREQUENCY DISCONTINUITY
1.27×10^{-4}	20	.7MHz	2.10MHz
1.27×10^{-4}	10	1.2MHz	2.80MHz
1.78×10^{-4}	40	.40MHz	.85MHz
1.78×10^{-4}	20	.40MHz	1.40MHz

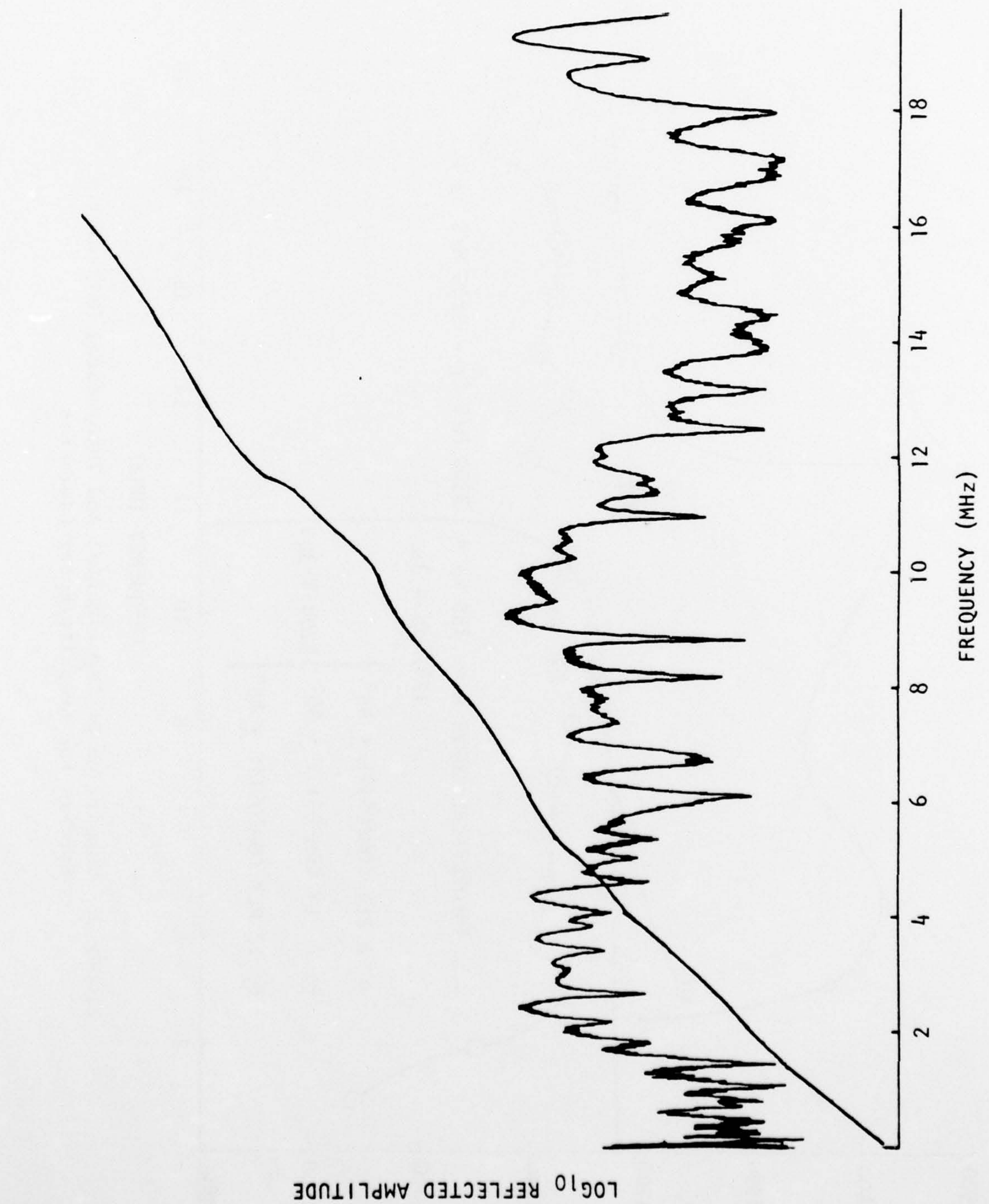

FIGURE 6. Ultrasonic Reflectance Spectrum Of 24 Ply 5208-T-300 Graphite Epoxy

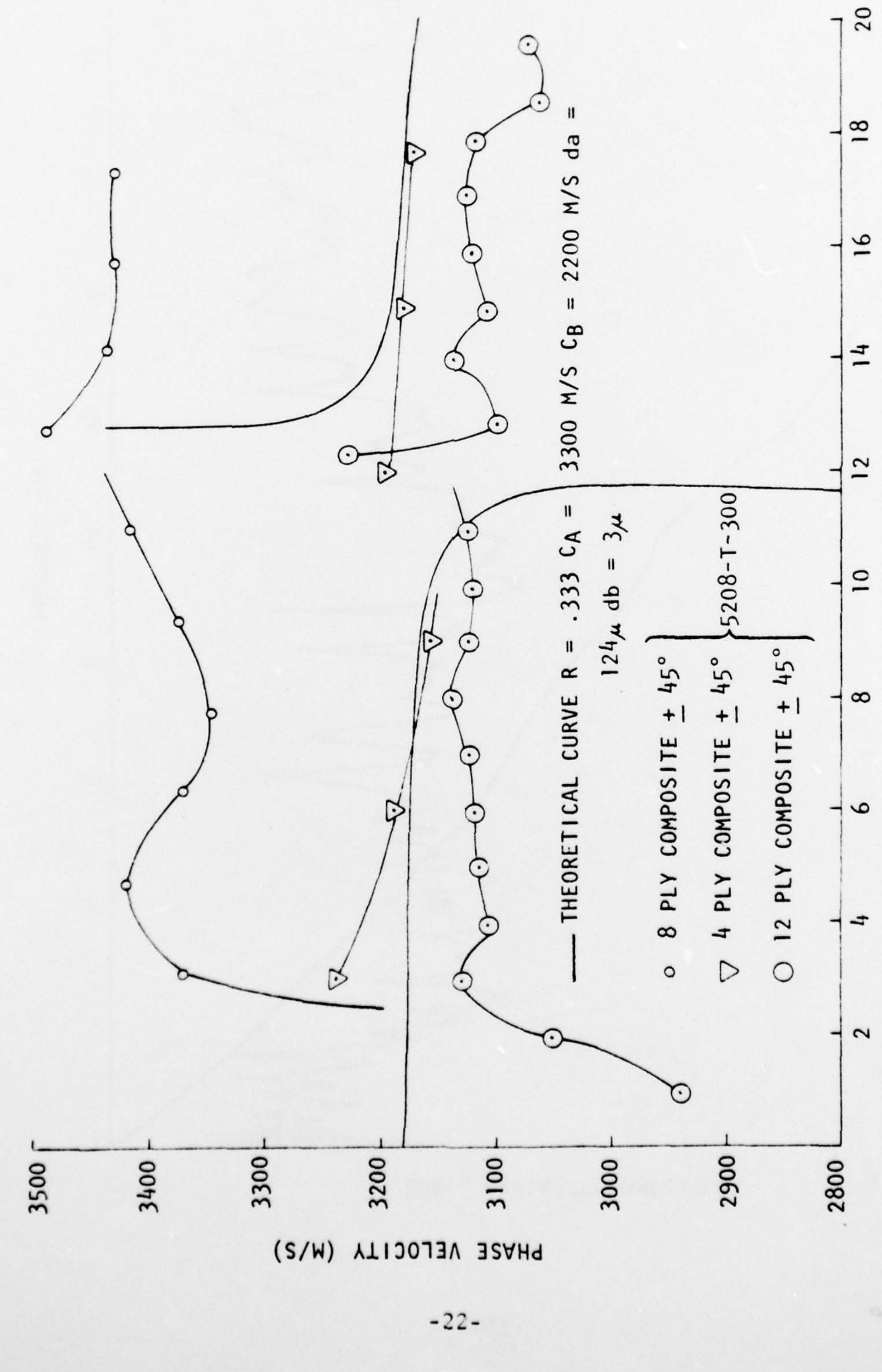

FIGURE 7. Comparison Of Experimental And Theoretical Reflectance Spectra For Graphite/Epoxy Laminates

TABLE III - AVERAGE PLY THICKNESSES FOR 5208-T-300 GRAPHITE/EPOXY LAMINATES

LAMINATE	TOTAL THICKNESS	AVERAGE PLY THICKNESS
4 PLY ± 45°	.021"	.0052
8 PLY ± 45°	.043"	.0054
12 PLY ± 45°	.062"	.0052
7 PLY ± 0°	.038"	.0054

For the materials with 6 mil nominal ply, thickness variation was large (>±10%) and the nominal value was used in velocity calculations.

velocity calculations for 5 mil graphite epoxy. Ply thicknesses for the $\pm$ 45° laminates are fairly uniform as can be seen in the micrograph in figure 8A.

Frequencies of thickness resonances were measured to about $\pm$ .04 MHz so that corresponding accuracies of velocity measurements ranged from $\pm$ 4% ($\pm$ 120 m/s) in the 1 MHz region to $\pm$ .4% ($\pm$ 12 m/s) near 10 MHz.

Comparison of Experimental and Theoretical Dispersion Relations

Figures 7, 9 and 10 show comparisons of theoretical dispersion relations (calculated from expressions (34) and (29)) with those determined from ultrasonic reflectance measurements. The values of parameters used for the analytical curves and their sources are summarized in table IV.

Examination of these experimental and theoretical data yields the following comparisons.

1. Locations of velocity discontinuities are predicted with reasonable accuracy.
2. Experimentally there is little or no indication of the predicted velocity decrease at frequencies just below a discontinuity.
3. The functional form of the dispersion relation just above a discontinuity is described reasonably well by theory but experimental results do not show as sharp a discontinuity as predicted.
4. Widths of discontinuities are rather difficult to accurately measure by these experiments however there is a definite indication of gaps of about the size predicted.
5. A rapid decrease in velocity below 4 MHz consistently appears in all the measurements and is too pronounced to be attributed to experimental error. It is probably a result of material dispersion related to viscoelastic damping.

A number of reasons can be hypothesized for the lack of agreement between theory and experiment, the least important of which is probably experimental error.

It is clear, from figure 7 that the velocity variation among samples of the same material can be significant and may be a function of laminate thickness. Such thickness dependent variations would of course contradict results for the model proposed.

This model, in fact, has a number of obvious deficiencies:

1. Effects of material dispersion and fiber scattering are ignored and these effects most probably contribute strongly to the decrease in velocity below 4MHz.
2. The assumption of a perfectly periodic structure can often be quite bad (this can be seen in the extreme case of figure 8B) Variations in ply thickness would undoubtedly produce the observed lack of sharpness in the velocity discontinuity; however, it is difficult to see why they would effect the

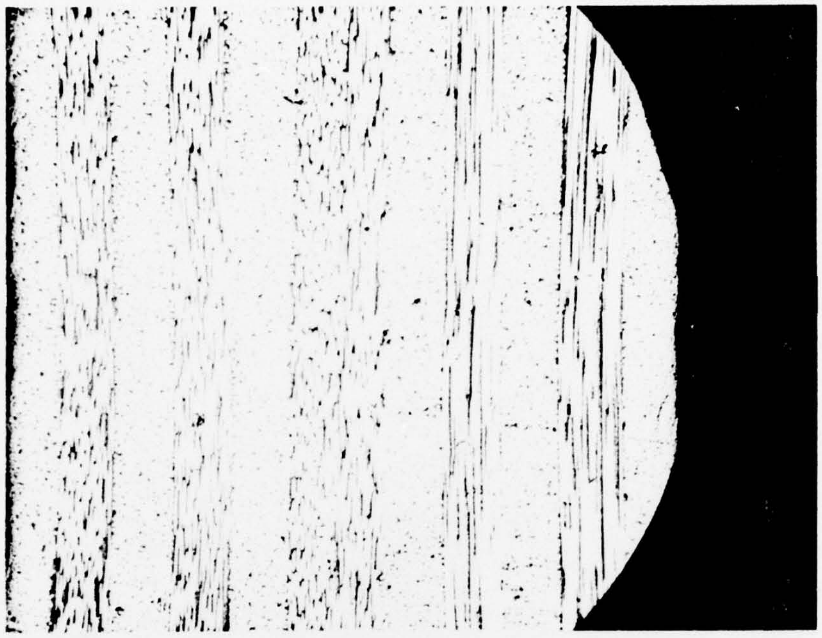

FIGURE 8A. Micrograph Of Cross Section Of 12 Ply Graphite Epoxy $\pm$ 45° Laminate (110x)

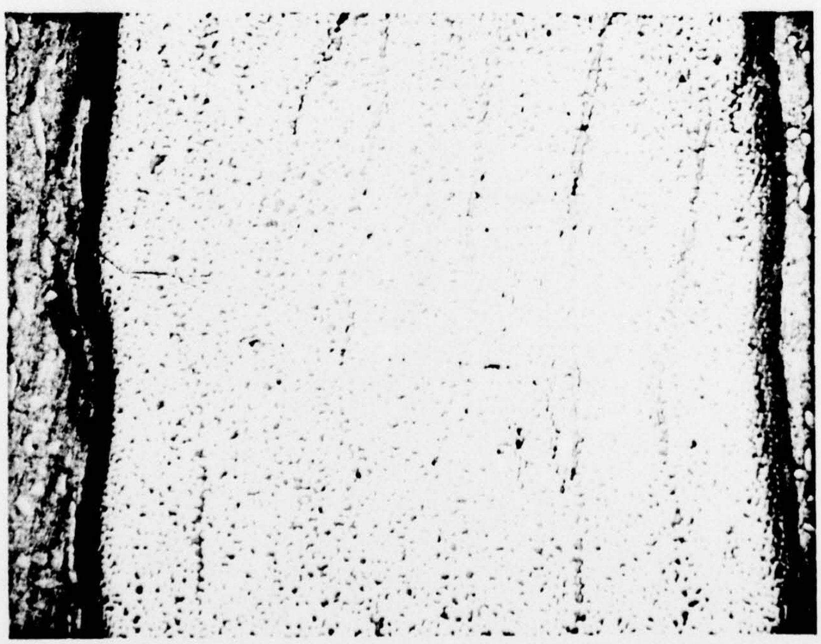

FIGURE 8B. Micrograph Of Cross Section Of 7 Ply 0° Laminate Graphite Epoxy (112x)

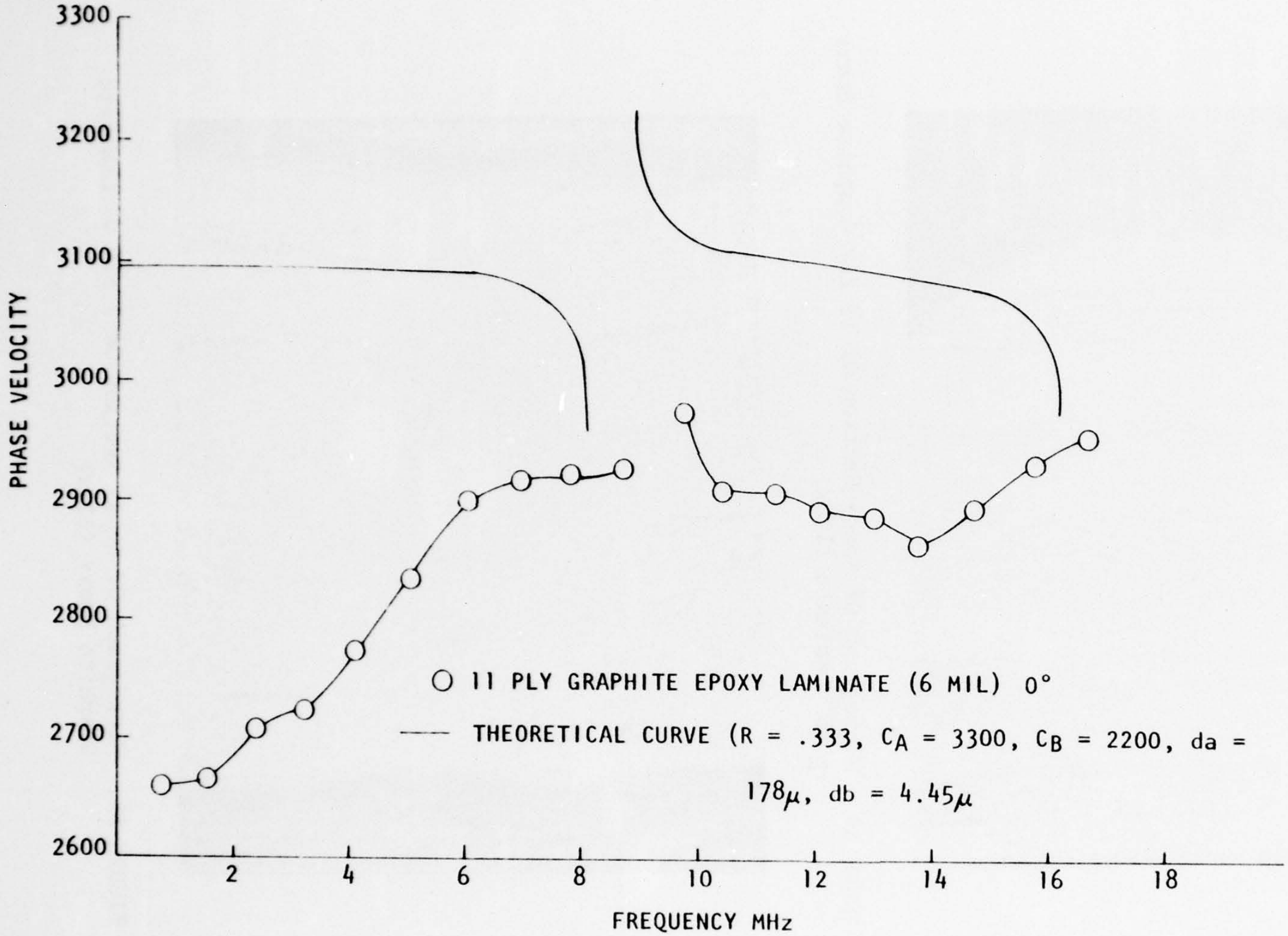

FIGURE 9. Comparison Of Experimental And Theoretical Reflectance Spectra For Graphite/Epoxy Laminates

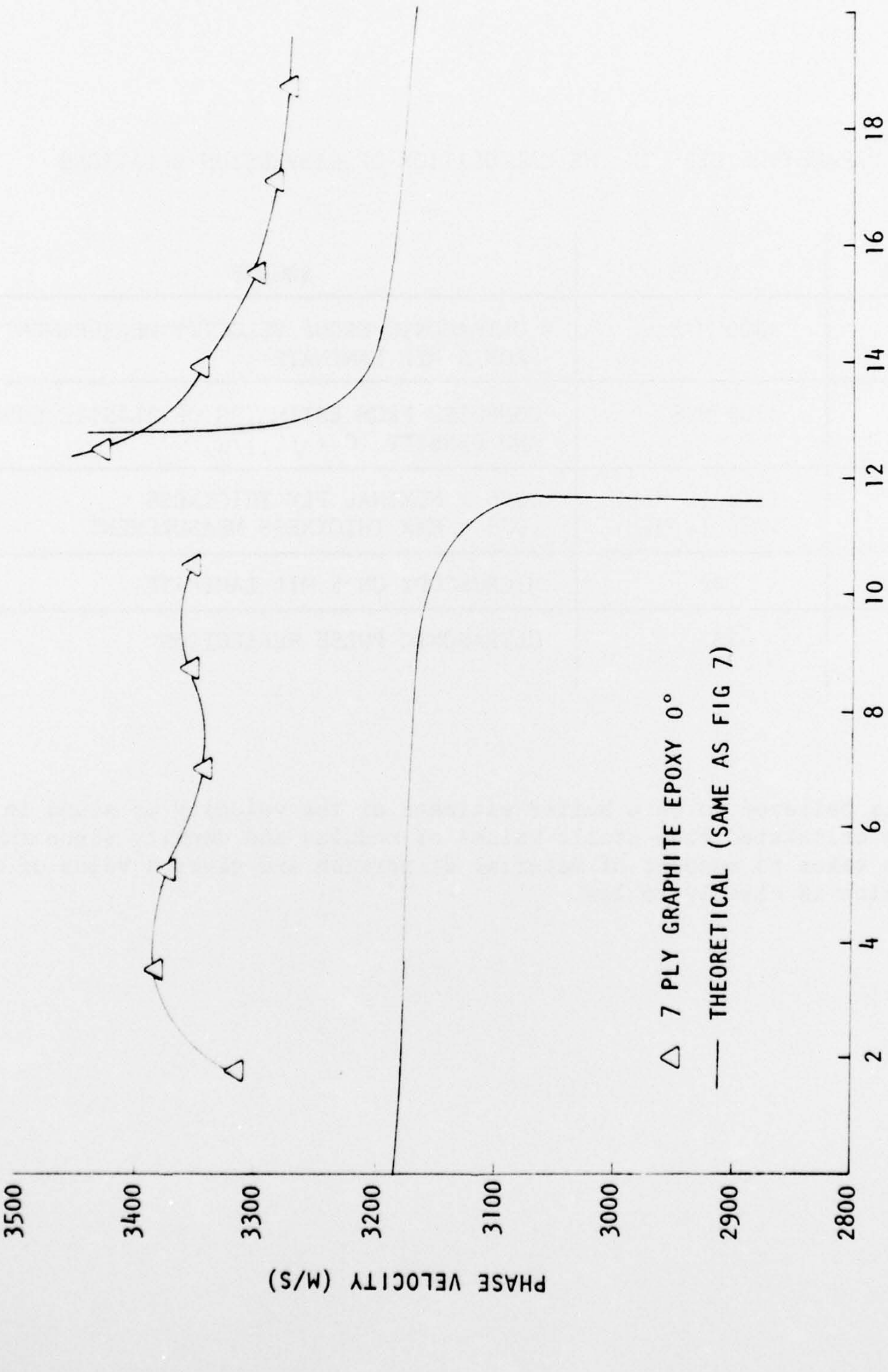

FIGURE 10. Comparison Of Experimental And Theoretical Reflectance Spectra For Graphite/Epoxy Laminates

TABLE IV - PARAMETERS USED IN THE CALCULATION OF DISPERSION RELATIONS

PARAMETER	VALUE	SOURCE
C_A	3300 M/S	† ULTRASONIC GROUP VELOCITY MEASUREMENT FROM 5 MIL LAMINATE
C_B	2200 M/S	COMPUTED FROM ESTIMATES OF ELASTIC CONSTANT AND DENSITY $C = \sqrt{C_{11}/\rho}$
da	124μ (5 MIL) 178μ (6 MIL)	.975 x NOMINAL PLY THICKNESS .975 x MAX THICKNESS MEASUREMENT
da/db	40	MICROSCOPY ON 5 MIL LAMINATE
R	.333	ULTRASONIC PULSE REFLECTION

† This is believed to be a better estimate of the velocity of sound in a layer than can be calculated from static values of modulus and density since the latter calculation takes no account of material dispersion and gives a value of about 2300 M/S which is clearly to low.

absence of a velocity dip which was predicted just below the discontinuity.

3. Along the same lines, the assumption of a periodic laminate implies the existence of a resin boundary layer of well defined thickness on the surface of the laminate. Although such a layer undoubtedly exists its thickness probably does not match the d_B for the internal boundary layer. Again this would imply that the concept of a material property, namely a thickness independent phase velocity may have limited validity for composites. This topic will be discussed further in the next section.

Variations Among Experimental Results

Measurements of sound velocity, in addition to differing from theory in functional form also fail to exhibit predicted consistency between laminates of different thicknesses. In principal curves for all three of the laminates in figure 7 should agree to within experimental error; however the 8 ply laminate is significantly different from the 4 ply and 12 ply composites, inspite of the fact that all three laminates were prepared together from the same starting material.

Even slight material variation can cause significant changes as is seen from figure 9 which is the dispersion relation for the same material as shown in figures 7 and 10 except with a nominal 6 mil ply thickness instead of the standard 5 mils. Clearly the functional form is significantly altered.

SUMMARY

Experimental and theoretical efforts applied to ultrasonic NDT of laminates have led to the following results:

- A convenient matrix algebra approach can be constructed for treating problems in ultrasonic, normal incidence, reflection spectroscopy.
- If the correct boundary conditions are imposed a thickness independent phase velocity can be defined and measured for periodic laminates in a manner analogous to that for monolithic materials.
- Predicted dispersion relations[1] for finitely thick laminates have discontinuities which are the same as those for infinite laminates.
- In the low frequency range (<3MHz) phase velocity is constant and dependent only on laminate composition and not on the thickness of specific laminae.
- Measured effects of geometric dispersion in graphite epoxy laminates are in semiquantitative agreement with predictions. Velocity discontinuities are on the order of 1 MHz both theorically and experimentally.
- Measurable variations in phase velocity are greatest at high frequencies (>8MHz) and are less that about 6%.
- Effects of geometric dispersion below the first discontinuity (∿8MHz) appear weak and material dispersion dominates resulting in variations in phase velocity up to 10%.
- Because of variability in interface and lamina thicknesses, many

graphite epoxy laminates are only a rough approximation of the idealized laminates modeled above. As a result material property scatter and thickness dependent material properties must be expected in real material.

CONCLUSIONS AND RECOMMENDATIONS

The use of spectrum analysis and the computation of dispersion relations is a very sensitive tool for interrogating laminates, in that effects from interfaces of microns in thicknesses can be revealed with acoustic waves of 12MHz and below. The formalism developed here may be useful in understanding such spectra in appropriate regimes.

For example the velocity of sound above the first frequency discontinuity could be used as a sensitive monitor of laminate uniformity and interface quality providing that a sufficiently high level of material uniformity could be achieved. The absence of this uniformity produces scatter in material properties and is the greatest obstacle to the efficient inspection in composite laminates.

Laminates of potentially high uniformity such as adhesively bonded sheet metal laminates are excellent candidates for inspection by the above techniques and should probably be considered in future studies. At low frequencies (i.e., wavelengths much longer than lamina thickness) static moduli can be related to velocity measurements by a simple rule of mixtures (40) which takes into account interface phase retardation. This technique may have application to directionally solidified eutectics.

Because material dispersion appears to have such a large effect at low frequencies (<3MHz) mechanisms for this dispersion such as viscoelastic damping should be studied.

The combined theoretical and experimental approach described here relates to discrete velocity measurements and therefore has limited resolution. Future investigations may require the use of continuous wave techniques in order to permit measurement of material properties continuously as a function of frequency.

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